Physics
John Dewey High School
Mr. Klimetz

Quantum Mechanics I

Derivation of Photon Momentum and the deBroglie Wavelength

The Newtonian expression for momentum (p) is given as

$p = mv$ [1]

where m is mass (kg) and v is velocity (m/s).

The energy of a photon (E_{photon}) as a consequence of Planck's photoelectric effect experiments can be calculated from the following expression:

$E_{photon} = hf$ [2]

where h is Planck's Constant (6.63×10^{-34} J-s) and f is photon frequency (Hz).

Additionally, the energy of a photon (E_{photon}) can also be expressed conceptually in terms of Einstein's equation of Energy-Mass equivalence

$E_{photon} = mc^2$ [3]

where E_{photon} = is the energy of the photon (J), m is the photon's "mass" (kg) and c is the speed of the photon in a vacuum (3.00×10^8 m/s).

Hence, we then set [2] and [3] as equal

$mc^2 = hf$

Dividing both sides by c^2 yields

$m = hf/c^2$ [4]

The speed of any wave phenomenon can be expressed as

$v_{wave} = f\lambda$ [5]

where f is wave frequency (Hz) and λ is wavelength (m).

Since photons travel in the form of a wave and possess a speed of c, where $v_{wave} = c = 3.00 \times 10^8$ m/s, we rewrite [5] as

$c = f\lambda$ [6]

Dividing both sides by λ and c yields

$1/\lambda = f/c$ [7]

Similarly, substituting c for v in [1] yields

$p_{photon} = mc$ [8]

Substituting [4] for m yields

$p_{photon} = hf/c^2 \times c = hf/c$ [9]

According to [7] $1/\lambda = f/c$. Then substituting [7] into [9] yields

$p_{photon} = h/\lambda$ [10]

the expression for the **momentum of a photon** [p_{photon}]

Dividing both sides of [10] by p_{photon} and multiplying both sides of [10] by λ yields

$\lambda = h/p$ [11]

the expression for the **deBroglie wavelength** [λ].